Indian Statistical Institute

Final Examination 2024-2025

Analysis of Several Variables, B.Math Second Year

Time : 3 Hours Date : 06.11.2024 Maximum Marks : 100 Instructor : Jaydeb Sarkar

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Q1. (10 marks) Consider two vectors $\langle a_{11}, a_{12}, a_{13}\rangle$ and $\langle b_{11}, b_{12}, b_{13}\rangle$ from $\mathbb{R}^3$, and assume that

$$\langle a_{11}, a_{12}, a_{13}\rangle \times \langle b_{11}, b_{12}, b_{13}\rangle \neq \langle 0, 0, 0\rangle.$$

Prove that the linear system of equations

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = 0 \\ b_{11}x + b_{12}y + b_{13}z = 0 \end{cases}$$

locally defines two of the variables as a function of the third.

Q2. (20 marks) Use Green's theorem to evaluate the line integral

$$\int_C (ye^{xy} + y)dx + (xe^{xy} + 3x)dy,$$

where C is the path from $(-2, 0)$ to $(2, 0)$ along the upper part of the ellipse $x^2 + 2y^2 = 4$.

Q3. (20 marks) Indicate whether the following statement is right or wrong, and justify your answer: The flux of

$$F = \frac{-1}{(x^2 + y^2 + z^2)^{\frac{3}{2}}}\langle x, y, z\rangle,$$

out of the unit sphere is zero.

Q4. (20 marks) Prove the divergence theorem for the special case where the surface is a box.

Q5. (20 marks) Use Stokes' theorem to evaluate

$$\int_C \langle -y^2, x, z^2\rangle \cdot dr,$$

where C is the curve of intersection of $y + z = 2$ and $x^2 + y^2 = 1$ oriented counterclockwise when viewed from the above.

Q6. (20 marks) For each $t > 0$, define

$$\Omega_t = \{(x, y) : t \leq x, y \leq 2t\}.$$

Define $f : (0, \infty) \longrightarrow (0, \infty)$ by

$$f(t) = \int_{\Omega_t} \exp\left(\frac{tx}{y^2}\right) dA.$$

Prove that there exists a constant α such that $f(t) = \alpha t^2$. Also determine the value of α.